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MODIFICATION OF GODUNOV'S NUMERICAL SCHEME FOR SOLVING PROBLEMS OF PULSED LOADING OF SOFT SOILS

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This paper considers a modification of the "predictor" stage of Godunov's discontinuity decay method for problems of nonlinear deformation of soft soils. Soil behavior is described by the Grigoryan's taking into account nonlinear diagrams of volumetric deformation, shear strength, and plasticity. Assumptions are formulated that reduce the auxiliary problem of the decay of a discontinuity in soils to the well-known problem that has a unique solution. Examples of numerical calculations are given.

Introduction. At present, numerical modeling of the unsteady deformation processes of soils has been performed using [1] the discontinuity decay method for solving gas-dynamic problems. Among the doubtless advantages of this method is the joint application of Lagrange's and Euler's approaches to description of the motion of continuous media, which considerably extends the range of problems solved. At the same time, for soils, there have been few papers on this topic that take into account their shear strength. This is due primarily to difficulties involved in solving the problem of decay of an arbitrary discontinuity. Godunov et al. [1], Kochin et al. [2], and Rozhdestvenskii and Yanenko [3] solved the discontinuity decay problem for media with better studied properties, in particular for fluids and gases, and constructed a generalized solution of the problem for a system of nonlinear equations. The studies cited showed that, generally, the discontinuity decay problem has a nonunique solution. Afanas'ev and Bazhenov [4] constructed discontinuous solutions of one-dimensional dynamic equations for elastoplastic media in Lagrange variables. Afanas'ev and Bazhenov [4] and Merzhievskii [5] studied a discontinuity decay model for one-dimensional viscoelastic and elastoplastic problems. Demidov and Koreneev [6] extended Godunov's finite volume method to the solution of two-dimensional problems of elastoplastic deformation of metals using a linearized version of calculation of the problem of decay of an arbitrary discontinuity. Abuzyarov et al. [7] used a combined approach to solving problems of unsteady deformation of compressible media, including soils, which takes into account the nonlinearity of the diagram of volume compression of material at large stresses. Demidov and Korneev [6] used a similar approach to solve problems at low stresses. In [6, 7], the constitutive relations are the Prandtl–Reuss plastic flow relations with the Mises yield condition and the laws of conservation of mass and momentum (Grigoryan model).

In the study of discontinuities solutions, the most widely used method for extending the constitutive equations of deformable media is the replacement of the original differential equations by an equivalent system of integral conservation laws. With this replacement, the system of equations is brought to divergent form and is integrated over an arbitrary spatial domain. As is shown in [8], the Prandtl–Reuss model cannot be reduced to divergent form, and, hence, transformed to a full system of integral conservation laws. Kondaurov [9, 10] considered a generalization of the dynamic hardening flow equations based on a kinematic equation in the form of a conservation law that relates the total displacement velocity gradient to particle velocity gradients. Using hyperbolic variational inequalities, Sadovskii [11, 12] examined the problem of constructing generalized solutions containing discontinuities of velocities, stress, and hardening parameters. The existence and uniqueness of a generalized solution are proved, and a classification of admissible discontinuous solutions of the type of elastoplastic waves is given.

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Despite the obvious advances, solution of the problem in the case of a nonlinear and irreversible diagram of volume compression of materials and possible shear strength and plasticity, as is assumed in the Grigoryan model, is a complex and still unsolved mathematical problem. In numerical implementation of the basic relations of the mechanics of continuous media and the Grigoryan model using Godunov's scheme of first-order accuracy, it is possible to ignore some of the indicated properties by virtue of high approximation viscosity.

In the present paper, we consider constitutive relations for deriving a solution of the problem of arbitrary discontinuity decay for soft soil or porous media taking into account the nonlinearity of diagrams of volume compression and shear strength. Assumptions are formulated that reduce the problem posed to a known one which has a unique self-similar solution. The validity of the adopted assumptions is examined and the applicability of the linearized method for calculating discontinuity decay in explosive loading problems for soil is studied over a wider pressure range than that indicated in [7].

1. Constitutive Equations. The problem of dynamic deformation of soil is formulated in Euler variables in Cartesian or cylindrical coordinates xy (y is the symmetry axis). The constitutive equations follow from the relations of continuum mechanics that express the laws of conservation of mass and momentum:

$$\rho_{,t} + (\rho u_x)_{,x} + (\rho u_y)_{,y} = -\nu(\rho u_x)/x,$$

$$(\rho u_x)_{,t} + (\rho u_x^2 - \sigma_{xx})_{,x} + (\rho u_x u_y - \sigma_{xy})_{,y} = -\nu(\rho u_x^2 - \sigma_{xx} - \sigma_{\theta\theta})/x, \tag{1.1}$$

$$(\rho u_y)_{,t} + (\rho u_x u_y - \sigma_{xy})_{,x} + (\rho u_x^2 - \sigma_{yy})_{,y} = -\nu(\rho u_x u_y - \sigma_{xy})/x.$$

Here ρ is the density, u_x and u_y are the velocity vector components in the x and y directions, respectively, σ_{ij} ($i, j = x, y, \theta$) are the Cauchy stress tensor components, and the subscript after the comma denotes partial differentiation with respect to the relevant variable; in the two-dimensional problem, $\nu = 0$, and in the axisymmetric problem, $\nu = 1$. The shear properties of soft soil are described using the Grigoryan model [13]

$$\frac{DS_{ij}}{dt} + \lambda S_{ij} = 2\mu e_{ij} \qquad (i, j = 1, 2), \tag{1.2}$$

where e_{ij} are the strain rate deviator components, S_{ij} are the stress deviator components, DS_{ij}/dt is the Jaumann derivative with respect to time, and μ is the shear modulus. If the plasticity condition is satisfied, $\lambda > 0$, and for purely elastic deformation, $\lambda = 0$. The system is closed by a barotropic equation which relates the pressure p and density: $p = f(\rho)$. Thus, system (1.1), (1.2) is closed relative to the vector of the unknowns $\{\rho, u_x, u_y, s_{xx}, s_{xy}, s_{yy}\}$.

2. Numerical Solution. Godunov's method for numerical solution of system (1.1), (1.2) and its modifications are described in detail in [6, 7]. In the present paper, we study the "predictor" stage of the well-known numerical method based on the general scheme given in [3]. The main constituents of the "predictor" stage are solution of the problem of decay of a discontinuity at each edge of the difference grid cell and determination of fluxes of the required quantities through relevant faces. The system of quasilinear equations for which the discontinuity decay problem is formulated in the present paper is a flat ($\nu = 0$), one-dimensional ($\partial/\partial z \equiv 0$), and elastic ($\lambda = 0$) analog of the full two-dimensional system (1.1), (1.2) in a coordinate system attached to the edge of the difference grid cell. A solution is constructed under the following assumptions:

1) the effect of shear waves on the parameters of longitudinal waves is insignificant;

2) discontinuous solutions (shock waves) are distinguished only for density and normal velocity.

Assumption 1 implies replacement of the Jaumann derivative in relations (1.2) by a total spatial derivative. This replacement is permissible for small rotations of the edges of the difference grid cells in a time step $\Delta t/2$ in view of the high approximation viscosity of the first-order scheme. In this case, the number of equations decreases by one because the last equation of (1.2) is integrated, resulting in the well-known relation for a uniaxial deformed state, $s_{zz} = -s_{rr}/2$. The required system of equations has the form

$$\dot{\rho} + u\rho_{,r} + \rho u_{,r} = 0, \quad \dot{u} + uu_{,r} - \sigma_{,r}/\rho = 0, \quad \dot{\sigma} + u\sigma_{,r} - \rho a^2 u_{,r} = 0,$$

$$\dot{v} + uv_{,r} - \tau_{,r}/\rho = 0, \qquad \dot{\tau} + u\tau_{,r} - \rho b^2 v_{,r} = 0. \tag{2.1}$$

We denote the vector of parameters $\{\rho, u, \sigma, v, \tau\}$ by $\boldsymbol{U}$; u and v are the velocity components and σ and τ are the stress tensor components in a coordinate system attached to the grid edge $r = r_0$. In this case, the discontinuity line r_0 is the boundary between the "left" cell of the difference grid with the parameters $\boldsymbol{U}_{(1)}$ and the "right" cell with the parameters $\boldsymbol{U}_{(2)}$:

$$r < r_0 \colon \quad \boldsymbol{U} = \boldsymbol{U}_{(1)}, \qquad r > r_0 \colon \quad \boldsymbol{U} = \boldsymbol{U}_{(2)}; \tag{2.2}$$

$$a^2 = c^2 + \frac{4\mu}{3\rho}, \qquad b^2 = \frac{\mu}{\rho}, \qquad c^2 = \frac{dp}{d\rho}. \tag{2.3}$$

The generalized solution of system (2.1) can have a finite number of piecewise smooth discontinuity lines (not more than five, according to number of characteristics), outside which it is a classical solution of the Cauchy problem:

$$\xi_1 = u - a, \qquad \xi_2 = u - b, \qquad \xi_3 = u, \qquad \xi_4 = u + b, \qquad \xi_5 = u + a. \tag{2.4}$$

The characteristics (2.4) on the plane (r, t) are straight lines and divide the characteristic plane into six zones. The zone numbers for which the solution is sought correspond to the characteristic numbers and are denoted by Roman numerals. To the left of the first characteristic and to the right of the fifth characteristic there are zones in which the variables coincide with the initial data (2.2). According to the theory of generalized solutions of the quasilinear equations [3], each characteristic $\xi = \xi_k(\boldsymbol{U})$ with number k can correspond to a centered rarefaction wave (RW) of the kth type, a shock wave (SW) of the kth type or a contact discontinuity (CD).

In the region of its smoothness, the self-similar solution $\boldsymbol{U}(y)$ of the quasilinear equations (2.1) reduces to solution of the system of ordinary differential equations

$$\frac{d\boldsymbol{U}}{dy} = \frac{\boldsymbol{r}^k(\boldsymbol{U})}{\boldsymbol{r}^k(\boldsymbol{U}) \operatorname{grad} \xi_k(\boldsymbol{U})}, \tag{2.5}$$

where $y = r/t$ is a self-similar variable and $\boldsymbol{r}^k(\boldsymbol{U})$ is the right eigenvector of the matrix of Eqs. (2.1); the operator grad is determined in [3]. The unknown quantities in zone I are denoted by capital letters with the superscript I: $R^{\rm I}$, $U^{\rm I}$, $V^{\rm I}$, $\Sigma^{\rm I}$, and $T^{\rm I}$. Using the first equation of (2.5), we eliminate the expression $\boldsymbol{r}^1(\boldsymbol{U}) \operatorname{grad} \xi_1(\boldsymbol{U})$ from the remaining differential equations and obtain a system of five equations, in which the independent variable is the density ρ:

$$\frac{dU}{d\rho} = -\frac{a(\rho)}{\rho} \quad \text{or} \quad U^{\rm I} = u_1 - \int\limits_{\rho_1}^{R^{\rm I}} \frac{a_1(\zeta)}{\zeta}\, d\zeta.$$

The normal stress on the discontinuity Σ can be written as $\Sigma = S - P$, where S and P are the deviator component and the pressure:

$$\frac{d\Sigma}{d\rho} = -a^2(\rho) \quad \text{or} \quad \Sigma^{\rm I} = \sigma_1 - \int\limits_{\rho_1}^{R^{\rm I}} a_1^2(\zeta)\, d\zeta. \tag{2.6}$$

Using expression (2.3) for the longitudinal wave velocity, expression (2.6) can be integrated:

$$\Sigma^{\rm I} = \sigma_1 - \int\limits_{\rho_1}^{R^{\rm I}} a_1^2(\zeta)\, d\zeta = s_1 - p_1 - \int\limits_{\rho_1}^{R^{\rm I}} a_1^2(\zeta)\, d\zeta = s_1 - p_1 - \int\limits_{\rho_1}^{R^{\rm I}} c_1^2(\zeta)\, d\zeta - \int\limits_{\rho_1}^{R^{\rm I}} \frac{4}{3}\, \frac{\mu_1}{\zeta}\, d\zeta$$
$$= s_1 - p_1 - p(R^{\rm I}) + p_1 - (4\mu_1/3) \ln{(R^{\rm I}/\rho_1)} = S^{\rm I} - P^{\rm I}.$$

Here

$$S^{\rm I} = s_1 - (4\mu_1/3) \ln{(R^{\rm I}/\rho_1)}. \tag{2.7}$$

The expression obtained here for the stress deviator component is also contained in [14, 15]. The shear velocity and stress components remain equal to the initial values in the "left" cell:

$$\frac{dV}{d\rho} = 0 \quad \text{or} \quad V^{\rm I} = v_1, \qquad \frac{dT}{d\rho} = 0 \quad \text{or} \quad T^{\rm I} = \tau_1.$$

In the case of SW propagation, to find the quantities in zone I, it is necessary to use relations on the discontinuity obtained from the integral analogs of the first two equations of system (2.1):

$$\oint \rho\, dr - \rho u\, dt = 0, \qquad \oint \rho u\, dr - (\rho u^2 - \sigma)\, dt = 0. \tag{2.8}$$

Denoting the rate of displacement of the discontinuity surface (or SW) in a normal direction by $D = dr/dt$ and integrating Eq. (2.8) over a contour enveloping the discontinuity surface, similarly to [1–3], we obtain the dynamic compatibility conditions at the discontinuities:

$$[\rho]D - [\rho u] = 0, \qquad [\rho u]D - [\rho u^2 - \sigma] = 0. \tag{2.9}$$

The square brackets denote the difference of the relevant quantities to the left and right of the discontinuity (relative to the normal vector). Using (2.9) for the quantities to the left and right of the characteristic ξ_1, we obtain the expression

$$U^{\rm I} = u_1 - \sqrt{(\sigma_1 - \Sigma^{\rm I})/(1/\rho_1 - 1/R^{\rm I})}.$$

The relationship between the pressure $P^{\rm I}$ and the density $R^{\rm I}$ is given by an equation of state. The deviator component $S^{\rm I}$ changes according to (2.7). Thus, two smooth curves representing the solution of Eqs. (2.5) and (2.9) in the space of variables $\boldsymbol{U} = \{R, U, \Sigma, V, T\}$ [3] pass through the point determined by relation (2.2). One curve represents a set of states that can be related to the state (2.2) by means of a RW of the kth type, and the other represents a set of states that can be related to the state (2.2) by means of a SW of the kth type. At the point given by relation (2.2), these two curves have second-order tangency, i.e, in a small neighborhood of the point given by relation (2.4), the shock transition curves are close to the curves describing transitions for rarefaction waves [3]. The choice of a segment of this or that curve as a solution is determined by the stability condition [3], which for RW in zone I is defined by the inequality $\xi_1(\boldsymbol{U}) > \xi_1(\boldsymbol{U}_{(1)})$. In view of the aforesaid, the velocity $U^{\rm I}$ and stress $\Sigma^{\rm I}$ in zone I are given by

$$U^{\rm I} = \begin{cases} u_1 - \displaystyle\int_{\rho_1}^{R^{\rm I}} \frac{a_1(\zeta)}{\zeta}\, d\zeta & \text{(RW)}, \\ u_1 - \sqrt{(\sigma_1 - \Sigma^{\rm I})/(1/\rho_1 - 1/R^{\rm I})} & \text{(SW)}, \end{cases}$$
$$\Sigma^{\rm I} = s_1 - P^{\rm I}(R^{\rm I}) - (4\mu_1/3)\ln(R^{\rm I}/\rho_1). \tag{2.10}$$

The formulas for the normal velocity $U^{\rm V}$ and stress $\Sigma^{\rm V}$ in zone V are obtained similarly.

The shear characteristics of the medium are determined in zones II and IV of the characteristic plane. Because the present Grigoryan's model assumes the absence of the dilatancy effect (i.e., density variation in the shear wave), the unknown quantities can be written as a function of density $\rho = \text{const} = R^{\rm I}$. Therefore, the relationship between shearing stresses and velocity is written as

$$T^{\rm II} - \tau_1 = \int_{v_1}^{V^{\rm II}} \rho b_1\, d\vartheta = R^{\rm I} b_1 (V^{\rm II} - v_1). \tag{2.11}$$

The effect of shear flow in zones II and IV on the normal components of the velocity and stress tensor is estimated as a quantity of the second order of smallness in strain components [16], and, therefore, as a first approximation, it can be ignored. Consideration of the linearized formulas [7] for U and Σ leads to a similar conclusion. The factor at u_i and σ_i has order $1/a_i$ and the factor at v_i and τ_i has order $1/a_i^2$ $(i = 1, 2)$.

The conditions on the third characteristic (on the CD)

$$U^{\rm II} = U^{\rm IV}, \qquad \Sigma^{\rm II} = \Sigma^{\rm IV}, \qquad T^{\rm II} = T^{\rm IV} = T, \qquad V^{\rm II} = V^{\rm IV} = V \tag{2.12}$$

make it possible to close expressions (2.10)–(2.12) and similar ones in other zones. We consider a configuration in the cell for which decay results in the formation of a RW propagating to the left and a SW propagating to the right. The condition on the CD yields the equation

$$u_2 - u_1 + \sqrt{\frac{\sigma_2 - \Sigma}{1/\rho_2 - 1/R^{\rm V}}} + \int_{\rho_1}^{R^{\rm I}} \frac{a_1(\zeta)}{\zeta}\, d\zeta = 0. \tag{2.13}$$

The densities as functions of Σ are obtained by solving the equations

$$\Sigma = s_1 - P^{\rm I}(R^{\rm I}) - (4\mu_1/3)\ln(R^{\rm I}/\rho_1), \qquad \Sigma = s_2 - P^{\rm V}(R^{\rm V}) - (4\mu_2/3)\ln(R^{\rm V}/\rho_2).$$

If in (2.13), we replace Σ by $-P$ and the longitudinal velocity a by the velocity of sound in the medium c (2.3), we obtain an expression that coincides with that given in [7] for calculation of discontinuity decay in a fluid medium with a nonlinear strain diagram. If the integral in (2.13) is replaced by an approximate expression using the "left"

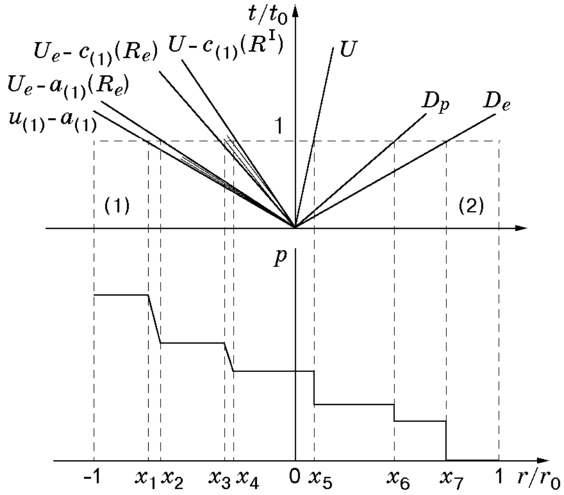


Fig. 1

TABLE 1

x	p, MPa	σ, MPa	u, m/sec
$-1 < x < x_1$	90	-100	0
$x_2 < x < x_3$	57.645	-47.645	60.36
$x_4 < x < x_5$	53.25	-43.25	70.80
$x_5 < x < x_6$	33.25	-43.25	70.80
$x_6 < x < x_7$	12.818	22.818	33.36
$x_7 < x < 1$	0	0	0

rectangle formula, the expression obtained coincides with the linearized formulas from [6, 7]. In addition, in the linearized version, RW and SW do not differ. Similarly, the tangential stress T and the velocity V in zones II and IV of the characteristic plane can be determined from the expressions

$$V = \frac{R^{\rm I} b^{\rm I} v_1 + R^{\rm V} b^{\rm V} v_2 + \tau_2 - \tau_1}{R^{\rm I} b^{\rm I} + R^{\rm V} b^{\rm V}}, \qquad T = \frac{R^{\rm I} b^{\rm I} R^{\rm V} b^{\rm V} (v_2 - v_1) + \tau_2 R^{\rm I} b^{\rm I} + \tau_1 R^{\rm V} b^{\rm V}}{R^{\rm I} b^{\rm I} + R^{\rm V} b^{\rm V}}. \tag{2.14}$$

Here $b^{\rm I} = b_1(R^{\rm I})$ and $b^{\rm V} = b_2(R^{\rm V})$. Expressions (2.14) are identical to those obtained in [7], except for the expressions for the densities $R^{\rm I}$ and $R^{\rm V}$. Thus, relations (2.10) and (2.14) allow one to evaluate the vector $\boldsymbol{U}$ over the entire characteristic plane (r, t).

Ignoring the effect of shearing stresses on the longitudinal wave characteristics, in the case of constant shear quantities in zones I and II, the parameters of longitudinal and shear waves can be calculated independently of each other. If $\{U, P, S, V, T\}$ are used as independent quantities, then, according to [11, 12], system (2.1) can be written as $L\boldsymbol{U} = 0$, where $L \equiv A\boldsymbol{U}_{,t} - B\boldsymbol{U}_{,r}$ is a differential operator (A and B are symmetrical coefficient matrices, and A is positively determined). From *a priori* estimates, Sadovskii [11, 12] inferred on the unique solubility of the discontinuity decay problem with nearly constant coefficients of the matrix A. Similar linearization was used in [6, 7] but the question of the uniqueness of the solution was not examined. Rozhdestvenskii and Yanenko [3] proved the existence and uniqueness of a generalized solution for a system of two hydrodynamic equations in the case where the stability conditions similar to those used above are satisfied and there are some restrictions on the dependence of pressure on density. Under assumptions 1 and 2, the first three equations of system (2.1) are analogs of the equations of gas dynamics, for which the discontinuity decay problem has been adequately studied.

Thus, using the first-order Godunov's scheme and the nonlinear Grigoryan's model of soil, a solution is constructed and justified for the problem of arbitrary discontinuity decay in the form of a combination of a SW and a RW for longitudinal waves and as a simple wave for transverse waves.

3. Numerical Calculations. Below we give results of calculations using a numerical procedure [7] with a modified "predictor" stage. In the calculations, we used the solution of the discontinuity decay problem obtained above taking into account nonlinear diagrams of volume compression and shear strength of the medium.

To estimate the applicability of the numerical procedure to solution of problems of elastoplastic deformation, we consider the following problem. The equation of state of the medium has the form $p = K(\rho/\rho_0 - 1)$, where $K = 250$ MPa; $\mu = 150$ MPa, $\sigma_{\rm y} = 15$ MPa, and $\rho_0 = 1$ g/cm^3. The parameters to the left of the discontinuity have the following values: $\rho_{(1)} = 1.36$ g/cm ^{3}, $p_{(1)} = p_0 = 90$ MPa, and $\sigma_{(1)} = \sigma_0 = -100$ MPa, and $u_{(1)} = 0$. The parameters to the right of the discontinuity correspond to the unperturbed state: $\rho_{(2)} = \rho_0$ and $p_{(2)} = \sigma_{(2)} = u_{(2)} = 0$. The "flow" configuration resulting from discontinuity decay is shown in Fig. 1. The figure shows a RW moving to the left and a CD and a SW moving to the right. The waves consist of an elastic precursor for both SW and SW and a plastic wave front. The time t is normalized to $t_0 = 1$ msec, and the space variable r is normalized to the dimension of the regions $r_0 = 1$ m. The sequence of the solution is shown schematically [12] at the bottom of

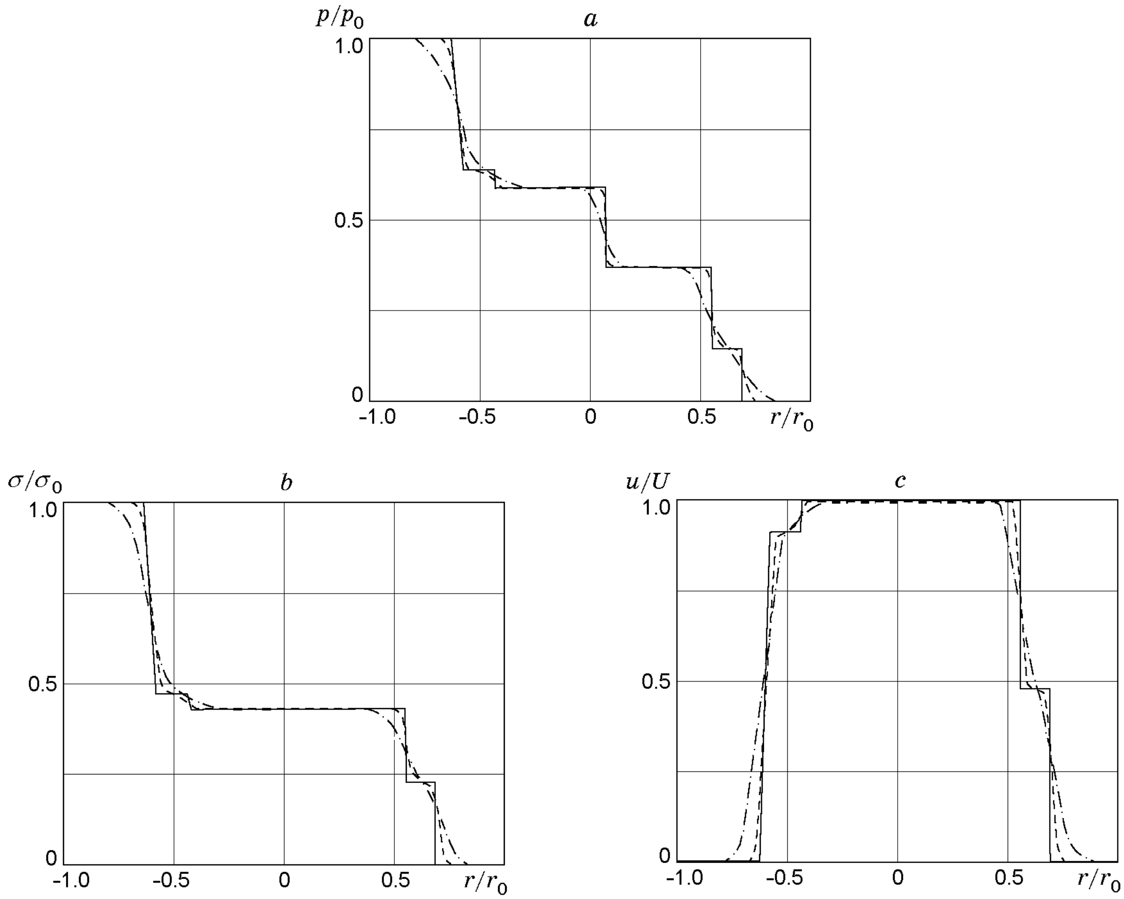
Fig. 1 for the pressure distribution as an example. The values of the dimensionless variable $x = r/r_0$ are as follows: $x_1 = -0.63$, $x_2 = -0.5787$, $x_3 = -0.4364$, $x_4 = -0.4292$, $x_5 = 0.0708$, $x_6 = 0.5524$, and $x_7 = 0.684$. The values of the pressure, stress, and velocity are listed in Table 1. In the ranges of the variable $x_1 < x < x_2$ and $x_3 < x < x_4$, a linear distribution is adopted. The distributions of pressure, stress, and velocity along the r axis are shown in Fig. 2 (solid curves). From Fig. 2a it follows that the pressure in elastic media undergoes a discontinuity. The stress and velocity are continuous for passage through the CD (Fig. 2b and c). To verify the validity of the above approach to solving the discontinuity decay problem in elastoplastic media, we obtained a solution of this problem by the "cross" scheme, which approximates the equations of flow theory in Lagrange variables. The solution was constructed without introducing artificial viscosity with Courant number equal to unity (relative to the rate of propagation of elastic longitudinal waves). The solution by the "cross" scheme is shown in Fig. 2 by dashed curves. A comparison of these two solutions shows the validity of the indicated approach. The dot-and-dashed curves correspond to the solution of the present problem by a scheme of first-order approximation. A comparison of the solutions shows that the assumption introduced within the framework of the first-order scheme is justified.

The effect of the nonlinear diagram on the wave pattern of the solution is examined in the problem of an explosion of a spherical explosive pressure charge in soil. Soft soils such as sand are characterized by nonlinear diagrams of compression and rarefaction and nonzero shear strength [17, 18]. The constants M and n of the power-law dependence are equal to 2.1 GPa and 1.8, respectively, and the constants A and B of the shock adiabat are equal to 500 m/sec and 2.41, respectively. The values of the constants of the interpolating polynomial are as follows: $\alpha = \beta = 0.06$, $\rho_1 = 1.86$ g/cm^3, $\rho_4 = 2.15$ g/cm^3, $\gamma_c = 3$, and $\gamma_p = 4$. The initial density of the sand mixture is $\rho_0 = 1.76$ g/cm^3, $\rho_g = 2.65$ g/cm^3, the initial velocity of sound for rarefaction is $C_0 = 350$ m/sec, the shear modulus is $G = 100$ MPa, and the yield strength constants are $k = 1.25$ and $b = 0.5$ MPa [17, 18]. The effect of the explosion is calculated using the model of an instantaneous wave detonation, and the initial pressure in the field of detonation products is equal to 12 GPa.

The time variations of the pressure on the contact boundary "detonation products–soil" are given in Fig. 3a (curves 1). In Figs. 3 and 4, solid curves show results of solution using nonlinear decay of a discontinuity, and dashed curves are obtained for linearized decay. The contact parameters are a solution of the relevant problem of discontinuity decay at the "predictor" stage of the numerical method. At the initial time, the difference is very large, but with time, it decreases considerably. Curves 2 in Fig. 3a correspond to the pressure in the cell adjoining the contact boundary. Figure 3b shows the time dependence of the velocity (notation same as in Fig. 3a). At a distance of about three initial radii of the charge, the difference practically disappears, whereas the pressure in the cells remains equal to about 150 MPa, which is far beyond the shear modulus and the yield strength of soft soils. The efficiency of the numerical procedure of [7] using the linearized version of calculation of discontinuity decay is also confirmed by the results of [17], in which such a solution is compared with experimental data for the problem of explosion of a laid-on explosive in sand.

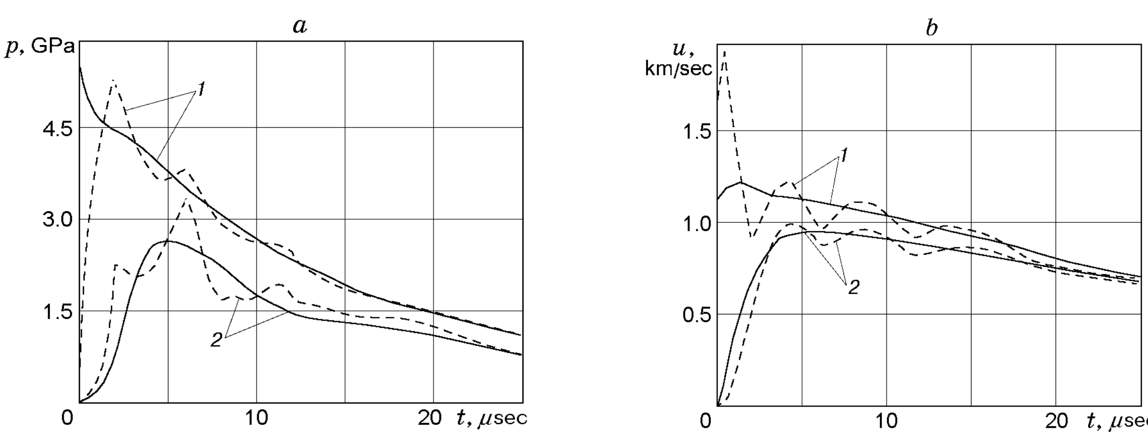
Below we give results of solving the problem of shock loading of soil using as an example the problem of collision of a long steel cylindrical impactor rod and a sandy soil [18]. The diameter of the impactor rod is 20 mm, and its length is 1000 mm. The mechanical characteristics of the impactor material are as follows: elastic modulus 200 GPa, Poisson's ratio 0.3, yield strength 1200 MPa, and density 7.8 g/cm^3. The experiments were conducted with a dry compressed sand mixture of a natural composition; the parameters of the equation of state of the mixture are given above.

A comparison of calculation results by the linearized and modified procedures with experimental data is given in Fig. 4. The time dependence of the resistance to the penetration of the impactor into the soil is shown. At the initial time $t = 0$, the ratio of the values of contact forces calculated by different procedures exceeds 300% but even at $t = 2$ μsec, the values of the forces become equal. The resistance force can be determined by integration of both contact stresses and stresses in the section of the impactor rod, as is assumed in the "inverse experiment" procedure [18]. The dot-and-dashed curve in Fig. 4 shows the resistance force obtained by integration of longitudinal stresses in the rod in the section at a distance of five diameters from the butt of the impactor. Considering scheme viscosity, there is no difference between the nonlinear and linearized versions of calculations. In calculations taking into account the nonlinear behavior of soil at the "predictor" stage of the numerical scheme, the maximum value of the force practically coincides with that obtained in experiment. The difference between experimental data (points in Fig. 4) and numerical results obtained by the linearized approach is 15%, which does not exceed the measurement error.



a
b
c
p/p_0
σ/σ_0
u/U
1.0
0.5
0
-1.0
-0.5
r/r_0

Fig. 2



a
b
p, GPa
u, km/sec
4.5
3.0
1.5
1.0
0.5
0
10
20
t, μsec
1
2

Fig. 3

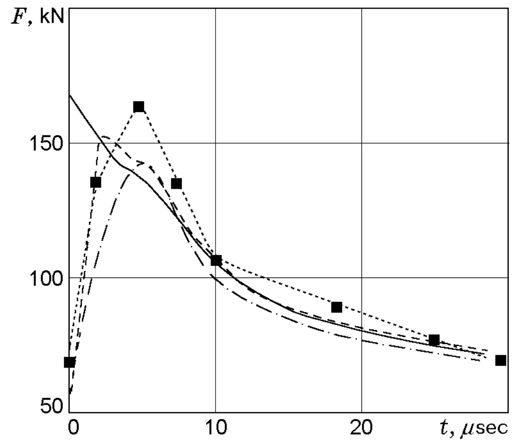


Fig. 4

4. Conclusions. A modification of Godunov's scheme for solving problems of nonlinear deformation of continuous media described by the Grigoryan model is obtained. Implementation of the "predictor" stage of the numerical scheme involves solution of the arbitrary discontinuity decay problem taking into account a nonlinear compression diagram and nonzero shear strength. A formulation of the problem is proposed that under certain assumptions reduces the problem to the well-known one which has a unique self-similar solution. Special cases of the discontinuity decay problem in this formulation are the well-known relations of fluid mechanics (in the absence of shear strength) and linearized relations, whose uniqueness can be proved using the method of variational inequalities. The assumptions for the first-order approximation scheme are validated by numerically solving test problems of pulsed deformation of soils. The linearized version of calculating the discontinuity decay problem at the "predictor" stage of the difference scheme gives acceptable results throughout the nearly entire range of pulsed loads. The nonlinear version used to validate the linearized approach allows estimating the parameters in the near zone of explosion or contact forces in solving collision problems with high gradients.

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